The Nominal Neural Test Model:

A Neural Test Model for Nominal Polytomous Data

SHOJIMA Kojiro[1]

OKUBO Tomoya[1,2]

ISHIZUKA Tomoichi[1]

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[1]Department of Test Analysis and Evaluation, Research Division, The National Center for University Entrance Examinations

[2]Research Fellow, Japan Society for the Promotion of Science

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Kojiro Shojima[1] Tomoya Okubo[1,2] Tomoichi Ishizuka[1]

Abstract

We developed the nominal neural test (NNT) model, which is a model of neural test theory (NTT) for analyzing nominal polytomous data. The NNT model is useful for evaluating the statistical characteristics of correct and incorrect choices of multiple-choice items. It becomes identical to the dichotomous NTT model when all items are binary. We report three analysis examples using the maximum likelihood (ML) method when the number of latent ranks Q equals 10 and 5, and the Bayesian method when $Q = 10$.

Key words: neural test theory, nominal neural test model, polytomous data, item category reference profile, latent rank theory.

名義ニューラルテストモデル: 多値の名義データのためのニューラルテストモデル

荘島宏二郎[1] 大久保智哉[1,2] 石塚智一[1]

要約

本研究では，多値の名義カテゴリデータを分析するためのニューラルテストモデルである名義ニューラルテスト (nominal neural test, NNT) モデルを提案した．NNT モデルは，多肢選択式問題において正答選択肢だけでなく誤答選択肢まで含めて分析するときに有効である．NNT モデルは，項目が 2 値のときには 2 値の NTT モデルと同一である．分析例では，潜在ランク数を 10 のもとでの最尤推定法，潜在ランク数が 5 のもとでの最尤推定法，潜在ランク数が 10 のときのベイズ推定法の結果を示した．

キーワード: ニューラルテスト理論，名義ニューラルテストモデル，多値データ，項目カテゴリ参照プロファイル，潜在ランク理論.

[1]Department of Test Analysis and Evaluation, Research Division, The National Center for University Entrance Examinations
[2]Research Fellow, Japan Society for the Promotion of Science

1 Introduction

Neural test theory (NTT; Shojima, 2007a) is a latent rank theory (LRT; Shojima, 2007f) for analyzing test data, and its mechanism is derived from the self-organizing map (SOM; Kohonen, 1995). The assumed latent scale in the NTT is rank-ordered, while the scale supposed in item response theory (IRT; e.g., Lord, 1980), which is the most prevailing test theory at present is continuous.

It is valid to install a rank-ordered scale in test theory because a test cannot distinguish two examinees who have nearly equal abilities. Although the abilities of examinees continuously vary, tests don't have high enough resolution to capture them. The most that a test can do is to grade examinees into several ranks.

Shojima (2007b) developed a graded neural test (GNT) model, which is a polytomous neural test model for analyzing ordered polytomous data, by extending the dichotomous neural test (DNT) model for binary data of true/false items. The GNT model reduces to the DNT model when all items are binary, and it is effective for analyzing testlet items composed of a few or several small questions and Likert-type (i.e., five-point agree-disagree scale) items of psychological questionnaires. However, polytomous data is not always ordered and sometimes nominal.

In fact, almost all true/false items are originally multiple-choice single-answer items, which are usually composed of one correct and a few wrong choices. That is, the data are originally nominal, and they are practically coded 1 (true) and 0 (false). Therefore, the DNT model can analyze multiple-choice items, although it cannot be used to obtain the information about incorrect choices.

However, it is important to analyze nominal data as they are. Such an analysis is necessary to clarify the variety of statistical characteristics of each nominal choice. For example, the analysis might reveal that an incorrect choice is inclined to be selected not only by the examinees with low ability but also by those with mid-level or even high ability. Or, the analysis can tell us which of two incorrect choices A and B is closer to the correct one. Such diagnostic knowledge is valuable educational information for teachers.

In IRT, the nominal categories model (Bock, 1972) is useful for dealing with nominal polytomous data, although the latent rank scale assumed in the model is continuous. The purpose of this study is to develop a polytomous model for analyzing nominal polytomous data, i.e., the nominal neural test (NNT) model, under the assumption that the latent scale is rank-ordered. The NNT model is a natural extension of the DNT model, and it reduces to the DNT model when the number of nominal categories is two.

2 Method

Let us assume that the sample size is N, the number of items is n, and that the response matrix of examinees is $\boldsymbol{X} = \{x_{ij}\}$ $(N \times n)$. The variable x_{ij} which is the response of examinee i to item j is

$$x_{ij} \in \{1, \cdots, C_j\}, \tag{1}$$

where C_j is the number of categories in item j, and each c $(= 1, \cdots, C_j)$ is the nominal category variable so that there is no quantitative relationship among their figures. In addition, $\boldsymbol{Z} = \{z_{ij}\}$ $(N \times n)$ is the missing indicator matrix (Shojima, 2007e), where z_{ij} is a dichotomous variable coded 1 when x_{ij} is observed and 0 when it is missing. Furthermore, let u_{ijc} be also a dichotomous variable coded 1 if $x_{ij} = c$ and 0 otherwise.

When the number of latent ranks is Q, the reference matrix of the nominal NTT model becomes

$$\boldsymbol{V} = \{v_{qjc}\} = \begin{bmatrix} v_{1,1,1} & \cdots & v_{1,1,C_1-1} & v_{1,2,1} & \cdots & v_{1,n,C_n-1} \\ v_{2,1,1} & \cdots & v_{2,1,C_1-1} & v_{2,2,1} & \cdots & v_{2,n,C_n-1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ v_{Q,1,1} & \cdots & v_{Q,1,C_1-1} & v_{Q,2,1} & \cdots & v_{Q,n,C_n-1} \end{bmatrix} \quad \Big\{Q \times \sum_{j=1}^{n}(C_j - 1)\Big\}, \tag{2}$$

where v_{qjc} is the probability that the examinee in latent rank R_q selects category c $(= 1, \cdots, C_j - 1)$ in item j. Each row vector in $\boldsymbol{V}$ is the rank reference vector (RRV), and each column vector is the item category reference profile (ICRP) of the corresponding item category. In addition, the expanded reference matrix becomes

$$\boldsymbol{P} = \{p_{qjc}\} = \begin{bmatrix} p_{1,1,1} & \cdots & p_{1,1,C_1} & p_{1,2,1} & \cdots & p_{1,n,C_n} \\ p_{2,1,1} & \cdots & p_{2,1,C_1} & p_{2,2,1} & \cdots & p_{2,n,C_n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ p_{Q,1,1} & \cdots & p_{Q,1,C_1} & p_{Q,2,1} & \cdots & p_{Q,n,C_n} \end{bmatrix} \quad \Big(Q \times \sum_{j=1}^{n} C_j\Big), \tag{3}$$

where p_{qjc} is the probability that the response of the examinee who belongs to latent rank R_q is c $(= 1, \cdots, C_j)$, and it is obtained by

$$p_{qjc} = v_{qjc} \qquad (c = 1, \cdots, C_j - 1), \tag{4}$$

$$p_{qjC_j} = 1 - \sum_{c=1}^{C_j-1} v_{qjc}. \tag{5}$$

The statistical learning procedure of the NNT model becomes as follows:

$$\text{For } (t{=}1;\ t \le T;\ t = t+1) \tag{6}$$

$$\text{— } \boldsymbol{X}^{(t)} \Leftarrow \text{Randomly sort the row vectors of } \boldsymbol{X}. \tag{7}$$

$$\quad \text{For } (h{=}1;\ h \le N;\ h = h+1) \tag{8}$$

$$\quad \text{— Obtain } \boldsymbol{z}_h^{(t)} \text{ and } \boldsymbol{u}_h^{(t)} \text{ from } \boldsymbol{x}_h^{(t)}. \tag{9}$$

$$\quad \text{— Select the winner node.} \tag{10}$$

$$\quad \text{— Obtain } \boldsymbol{V}^{(t,h)} \text{ by updating } \boldsymbol{V}^{(t,h-1)}. \tag{11}$$

$$\quad \text{— Obtain } \boldsymbol{P}^{(t,h)} \text{ from } \boldsymbol{V}^{(t,h)}. \tag{12}$$

$$\text{— } \boldsymbol{V}^{(t+1,0)} \Leftarrow \boldsymbol{V}^{(t,N)}. \tag{13}$$

$$\text{— } \boldsymbol{P}^{(t+1,0)} \Leftarrow \boldsymbol{P}^{(t,N)}. \tag{14}$$

where $\boldsymbol{x}_h^{(t)}$ is the h-th row vector of $\boldsymbol{X}^{(t)}$ that is the input data at the t-th period. In addition, $\boldsymbol{z}_h^{(t)} = \{z_{hj}^{(t)}\}$ $(n \times 1)$ is the missing indicator vector for $\boldsymbol{x}_h^{(t)}$, and $\boldsymbol{u}_h^{(t)} = \{u_{hjc}^{(t)}\}$ $(\sum_j C_j \times 1)$ is the binary variable vector corresponding to $\boldsymbol{x}_h^{(t)}$. Furthermore, $\boldsymbol{V}^{(t,h)}$ is the updated reference matrix after learning input vector $\boldsymbol{x}_h^{(t)}$, and $\boldsymbol{P}^{(h,t)}$ is the expanded reference matrix obtained from $\boldsymbol{V}^{(h,t)}$. The recommended initial value for v_{qjc} is $q/(Q+1)$ when the item category is the correct answer and $(Q-q+1)/\{(C_j-1)(Q+1)\}$ when it is the incorrect choice.

Line (7) is necessary for canceling the order effect of the input data on the statistical learning. The winner node in Line (10) can be determined from the Bayesian method (Shojima, 2007g) as follows:

$$R_w:\ \ w = \arg\max_{q \in Q} \bigl\{ \ln p(\boldsymbol{u}_h^{(t)} | \boldsymbol{p}_q^{(t,h-1)}) + \ln \pi_q \bigr\}, \tag{15}$$

where π_q is the prior probability that the winner node is latent rank R_q, and the first term in the parenthes in the above equation is the likelihood . That is,

$$p(\boldsymbol{u}_h^{(t)} | \boldsymbol{p}_q^{(t,h-1)}) = \prod_{j=1}^{n} \prod_{c=1}^{C_j} \bigl(p_{qjc}^{(t,h-1)}\bigr)^{z_{hj}^{(t)} u_{hjc}^{(t)}}. \tag{16}$$

Equation (15) is identical to the maximum likelihood (ML) method (Shojima, 2007c) when the prior probability is not assumed.

In (11), the reference matrix is updated by

$$\boldsymbol{V}^{(t,h)} = \boldsymbol{V}^{(t,h-1)} + (\boldsymbol{h}^{(t)} \mathbf{1}_n') \odot (\mathbf{1}_Q \boldsymbol{z}_h^{(t)\prime}) \odot (\mathbf{1}_Q \boldsymbol{u}_h^{(t)\prime} - \boldsymbol{V}^{(t,h-1)}), \tag{17}$$

where

$$\boldsymbol{h}^{(t)} = \left\{ h_{qw}^{(t)} \middle| h_{qw}^{(t)} = \alpha_t \exp\left(-\frac{(q-w)^2}{2\sigma_t^2}\right) \right\} \quad (Q \times 1), \tag{18}$$

$$\alpha_t = \frac{T-t+1}{T}\alpha_1, \tag{19}$$

and

$$\sigma_t = \frac{(T-t)\sigma_1 + (t-1)\sigma_0}{T-1}. \tag{20}$$

The factor $h_{qw}^{(t)}$, called "tension", controls the learning size of the RRV of latent rank R_q. It regulates geographically closer nodes to the winner node to have greater size of updating their RRVs. The constant α determines the size of the tension, and it linearly decreases from α_1 as t increases. In addition, σ specifies the region where the learning propagates, and it also reduces from σ_1 to σ_0 as t approaches T.

From the estimate of the expanded reference matrix $\hat{\boldsymbol{P}}$ calculated from the estimate of the reference matrix $\hat{\boldsymbol{V}}$, the test reference profile (TRP; Shojima, 2007a) is given by

$$\boldsymbol{t} = \left\{ t_q \middle| t_q = \sum_{j=1}^{n} \sum_{c=1}^{C_j} w_{jc}\hat{p}_{qjc} \right\} \quad (Q \times 1), \tag{21}$$

where w_{jc} is the weight for item j when the correct choice of item j is category c.

The estimation of the latent rank for each examinee is identical to the winner node selection method. That is, the latent rank of examinee i, R_{r_i}, is estimated as follows:

$$R_{r_i}: \; r_i = \arg\max_{q \in Q}\{\ln p(\boldsymbol{u}_i|\hat{\boldsymbol{p}}_q) + \ln \pi_q\}, \tag{22}$$

where

$$p(\boldsymbol{u}_i|\hat{\boldsymbol{p}}_q) = \prod_{j=1}^{n}\prod_{c=1}^{C_j}(\hat{p}_{qjc})^{z_{ij}u_{ijc}}. \tag{23}$$

The maximum a posteriori (MAP) rank is obtained by the above equation if the prior probability is supposed, and the maximum likelihood (ML) rank is estimated when no prior probability is imposed. The latent rank distribution (LRD; Shojima, 2007a) is then

$$\boldsymbol{f} = \left\{ f_q \middle| f_q = \sum_{i=1}^{N} f_{iq} \right\} \quad (Q \times 1), \tag{24}$$

where f_{iq} is a dichotomous variable coded 1 if the latent rank of examinee i is R_q and 0 otherwise.

In addition, the rank membership profile (RMP; Shojima, 2007c), which is useful for reviewing the probabilities of each examinee's belonging to respective ranks, and the posterior RMP (Shojima, 2007g) are given by

$$\boldsymbol{p}_i^{(ML)} = \left\{ p_{iq}^{(ML)} \middle| p_{iq}^{(ML)} = \frac{p(\boldsymbol{u}_i|\hat{\boldsymbol{p}}_q)}{\sum_{q'=1}^{Q} p(\boldsymbol{u}_i|\hat{\boldsymbol{p}}_{q'})} \right\} \quad (Q \times 1), \tag{25}$$

and

$$\boldsymbol{p}_i^{(MAP)} = \left\{ p_{iq}^{(MAP)} \middle| p_{iq}^{(MAP)} = \frac{p(\boldsymbol{u}_i|\hat{\boldsymbol{p}}_q)\pi_q}{\sum_{q'=1}^{Q} p(\boldsymbol{u}_i|\hat{\boldsymbol{p}}_{q'})\pi_{q'}} \right\} \quad (Q \times 1). \tag{26}$$

From the RMPs and the posterior RMPs of all the examinees, the rank membership distribution (RMD; Shojima, 2007c) and the posterior RMD (Shojima, 2007g) are then obtained by

$$\boldsymbol{g}^{(ML)} = \left\{ g_q^{(ML)} \middle| g_q^{(ML)} = \sum_{i=1}^{N} p_{iq}^{(ML)} \right\} \quad (Q \times 1) \tag{27}$$

and

$$\boldsymbol{g}^{(MAP)} = \left\{ g_q^{(MAP)} \middle| g_q^{(MAP)} = \sum_{i=1}^{N} p_{iq}^{(MAP)} \right\} \quad (Q \times 1). \tag{28}$$

Finally, the observation ratio profile (ORP; Shojima, 2007e) expresses each item's observed/missing response ratio through the latent rank scale. The unweighted and weighted ORPs and the posterior weighted ORP are obtained by

$$\boldsymbol{z}_j^{(U)} = \left\{ z_{qj}^{(U)} \middle| z_{qj}^{(U)} = \frac{\sum_{i=1}^{N} z_{ij} f_{iq}}{\sum_{i=1}^{N} f_{iq}} \right\} \quad (Q \times 1) \tag{29}$$

$$\boldsymbol{z}_j^{(W)} = \left\{ z_{qj}^{(W)} \middle| z_{qj}^{(W)} = \frac{\sum_{i=1}^{N} z_{ij} p_{iq}^{(ML)}}{\sum_{i=1}^{N} p_{iq}^{(ML)}} \right\} \quad (Q \times 1), \tag{30}$$

and

$$\boldsymbol{z}_j^{(PW)} = \left\{ z_{qj}^{(PW)} \middle| z_{qj}^{(PW)} = \frac{\sum_{i=1}^{N} z_{ij} p_{iq}^{(MAP)}}{\sum_{i=1}^{N} p_{iq}^{(MAP)}} \right\} \quad (Q \times 1), \tag{31}$$

respectively.

3 Analysis

3.1 Example 1

We analyzed data of a world history test. The sample size was 2,049, and the number of items was 36. All items were multiple-choice single-answer items, and the number of categories of each item was four or six. However, the categories for which selection ratios are less than 10% were merged into category x.

This section describes the result of the ML method. The ML method is identical to the Bayesian method when the prior distribution is uniform. The parameters necessary for the statistical learning were determined as $(Q, T, \alpha, \sigma_1, \sigma_0) = (10, 500, 0.1, 10, 1)$.

Figure 1 shows the item category reference profiles (ICRPs) of the 36 items. Figure 2 shows the test reference profile (TRP), the latent rank distribution (LRD), the rank membership distribution (RMD), the scatter plot of the scores and the estimated ML ranks, and the rank membership profiles (RMPs) of examinees 1–16, respectively. In Figure 1, the ICRP with an asterisk in each panel is the profile of the correct choice. In general, the ICRPs of the correct choices have a tendency to increase as the latent rank becomes higher. Although not every ICRP of the correct answer monotonically increases, a constraint to make it monotonically increase can be added to the statistical learning process, as shown by Shojima (2007a, 2007b). The nominal NTT model is effective for evaluating the behavior of the incorrect choices. For example, the ICRP of category 3 of item 6 clearly shows that the category is an attractive incorrect choice for lower rank examinees. In addition, item 15 is virtually a two-alternative item even though the item originally has four categories.

Figure 2(a) shows that the TRP is monotonically increasing; nevertheless, the ICRPs of all items do not always monotonically increase. That is, the scale assumed under the NTT is rank-ordered in terms of the TRP. The NTT is a latent rank theory so that it becomes self-contradictory unless the TRP is monotonically increasing. The possibility that the TRP does not monotonically increase becomes large as the number of latent ranks, Q, becomes larger. Therefore, it is recommended that Q be not more than ten to ensure that the TRP is monotonic, and, logically speaking, Q does not become large because the most that a test can do is to grade examinees into several ranks.

As indicated by Shojima (2007a, 2007b, 2007c), the LRD and RMD (Figures 2(b) and 2(c)) shows that the frequencies of the latent ranks at both ends of the scale (R_1 and R_{10}) are larger than those of the intermediate ranks, and this tendency is derived from the SOM. The scatter plot in Figure 2(d) shows that the latent ranks of the examinees with the same

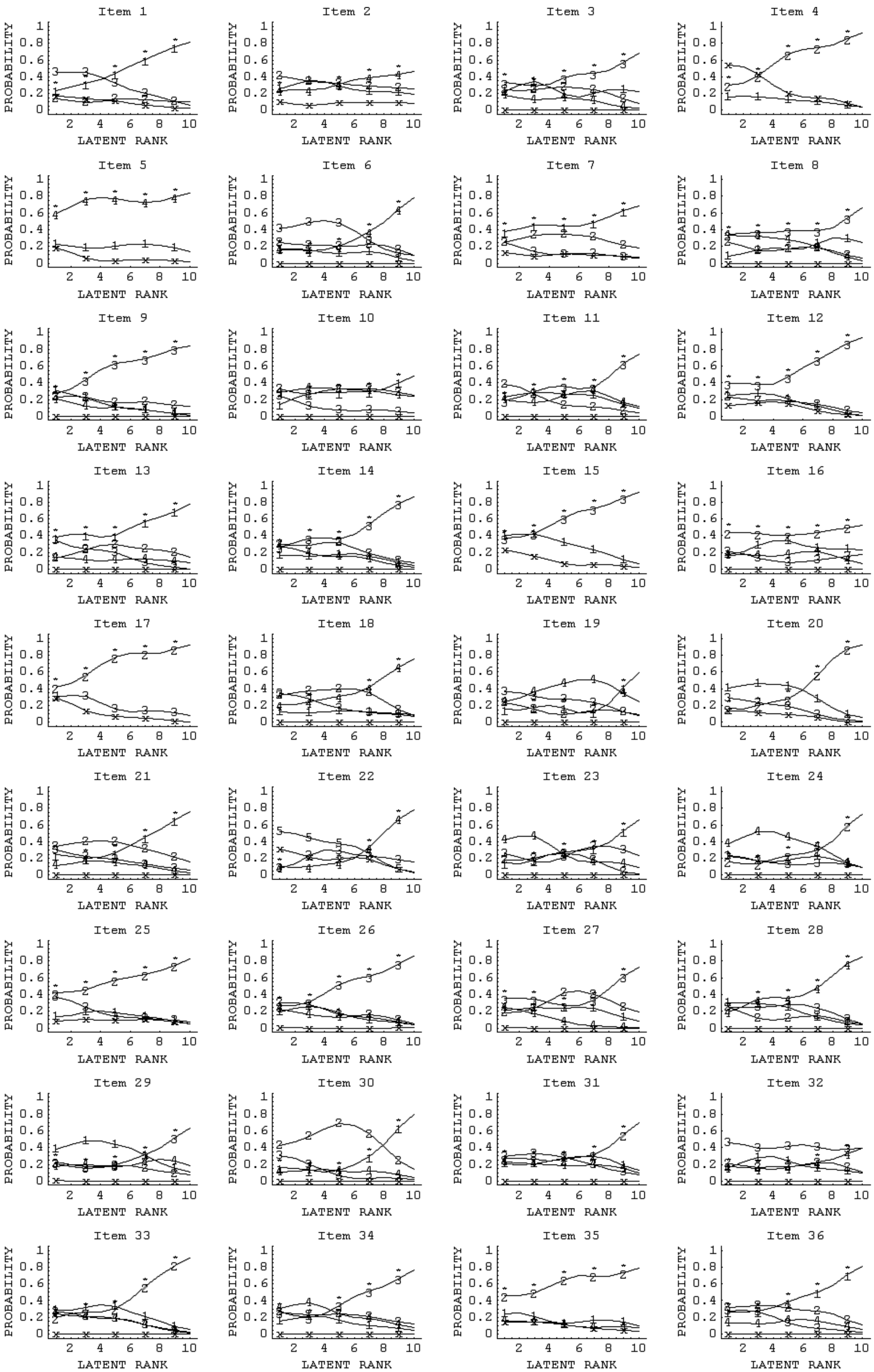
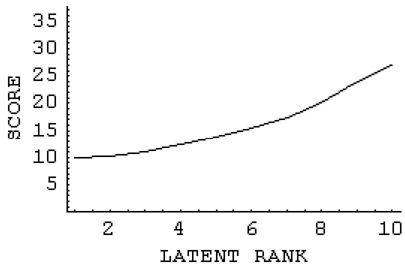
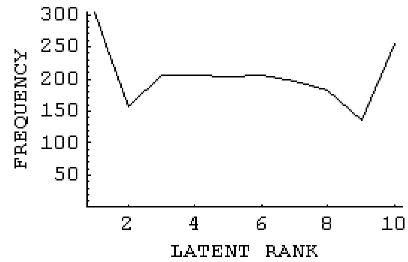


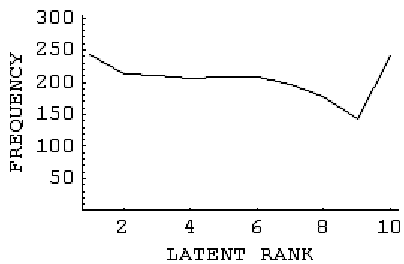
Figure 1: Item Category Reference Profiles (ML)



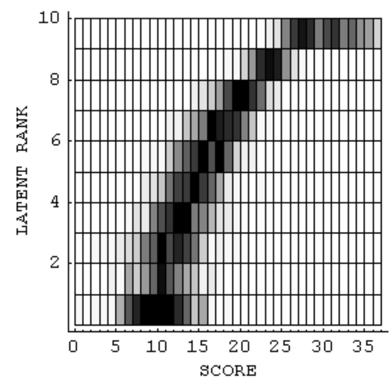
(a) Test Reference Profile (ML)



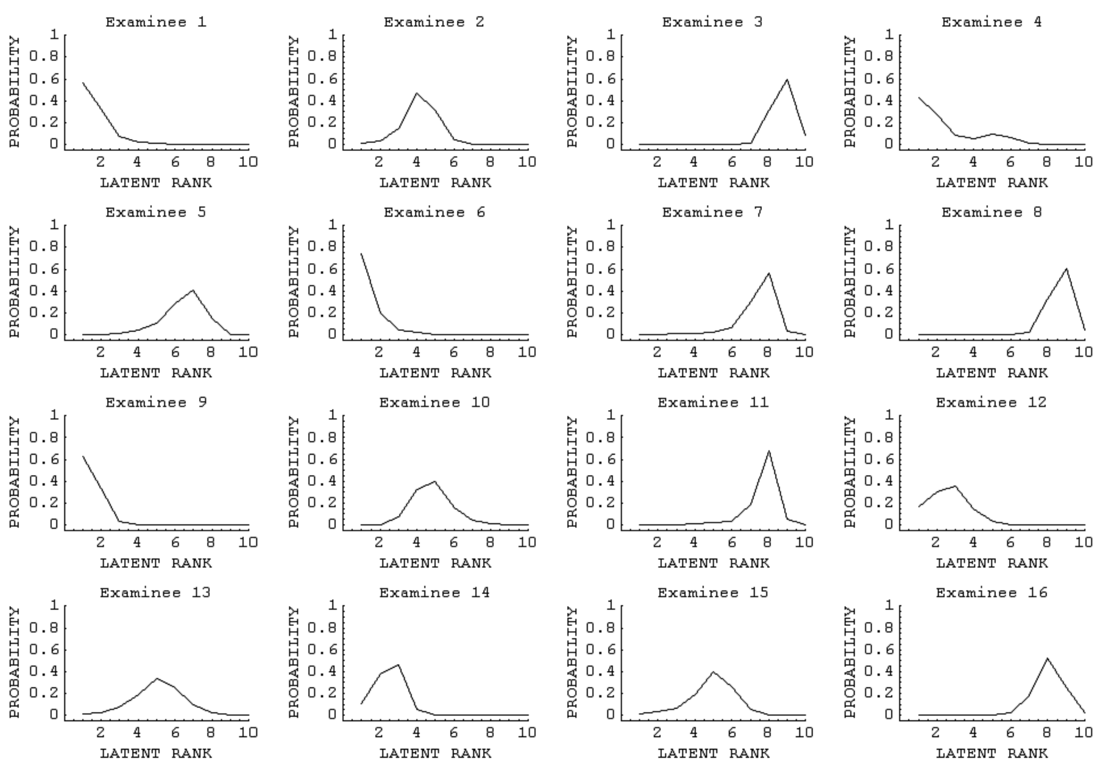
(b) Latent Rank Distribution (ML)



(c) Rank Membership Distribution (ML)



(d) Scatter Plot of Scores and Ranks (ML)



(e) Rank Membership Profiles of Examinees 1–16 (ML)

Figure 2: TRP, LRD, RMD, Scatter Plot, and RMPs given by ML Method

score are not always estimated to be identical. In addition, it is clear from Figure 2(e) the probability sizes of the examinees with the same estimated rank become different when their response patterns are different. For example, the certainty that examinee 10 belongs to latent rank R_5 is stronger than for examinee 13.

3.2 Example 2

This section shows the result analyzed when the number of latent ranks is five. There are many teachers who believe that the maximum number of ranks is five when they give meaningful and substantial labels to the ranks. Therefore, it is worthwhile to report the result when Q is five, provided that the other parameters are the same as those set in Example 1. Figure 3 shows the ICRPs of the 36 items, and Figure 4 shows the TRP, LRD, RMD, the scatter plot of the scores and ranks, and the RMPs of examinees 1–16.

It is obvious from the difference between Figures 2 and 3 that ICRPs are inclined to be monotonically increasing. In addition, the frequencies at both ends of the latent scale (R_1 and R_5) are larger than those of the intermediate ranks, as observed in Example 1.

3.3 Example 3

In Examples 1 and 2, the frequencies of the latent ranks at both ends are inclined to be larger. However, there are some teachers and test administrators who want to grade examinees into latent ranks with nearly equal frequencies. Therefore, the result of the Bayesian method for $Q = 10$ is shown in this section. The prior distribution used in the analysis is

$$\pi_q = \begin{cases} 0.08500 & q = 1, 10 \\ 0.10375 & q = 2, \cdots, 9, \end{cases} \tag{32}$$

and the other conditions are identical to those in Example 1.

Figure 5 shows the ICRPs of the 36 items, and Figure 6 shows the TRP, the posterior LRD, the posterior RMD, the scatter plot of the scores and the MAP ranks, and the posterior RMPs of examinees 1–16. It is clear from the posterior LRD and RMD (Figures 6(b) and 6(c)) that the frequencies of the latent ranks at both ends of the scale are lower than the ones given by the ML method (Figures 2(b) and 2(b)). This effect is promoted when a stronger prior distribution is selected. In addition, the probabilities of the latent ranks at both ends of the scale are lower in the posterior RMPs (Figure 6(d)) as compared to the RMPs given by the ML method (Figure 2(d)). For example, the probability that examinee 1 belongs to

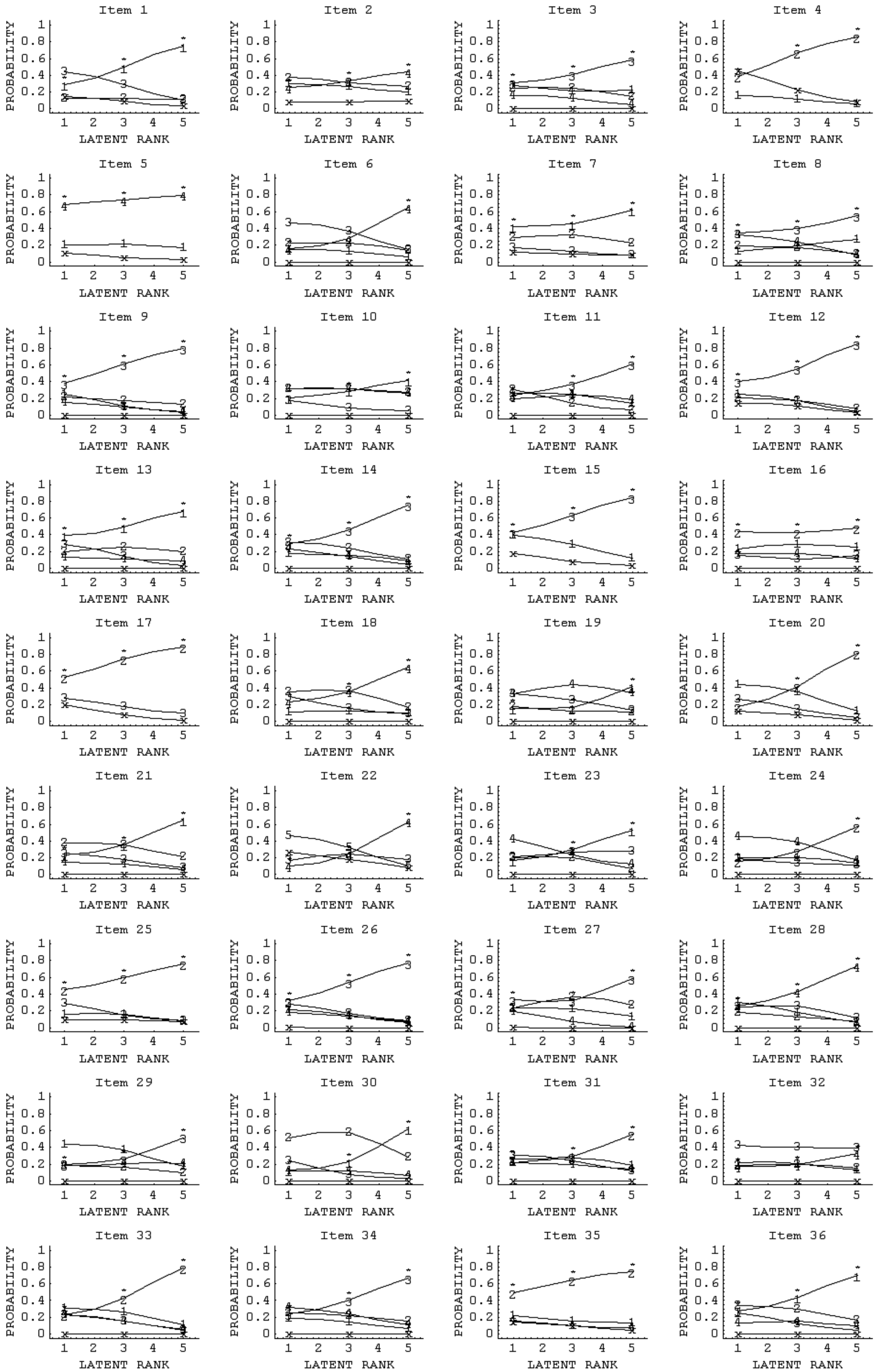
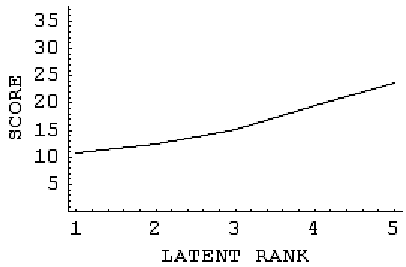
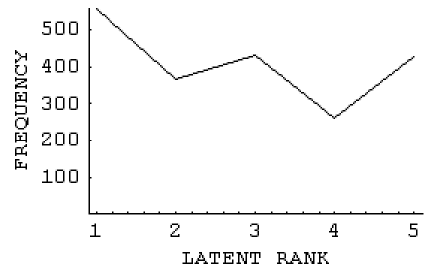


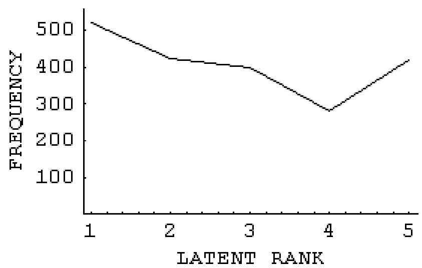
Figure 3: Item Reference Profiles ($Q = 5$)



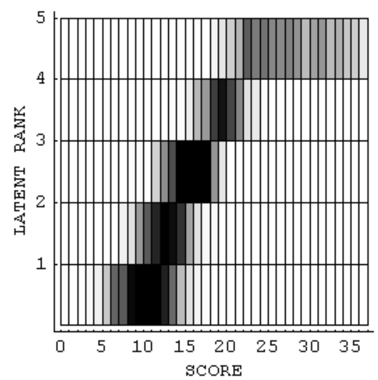
(a) TRP ($Q = 5$)



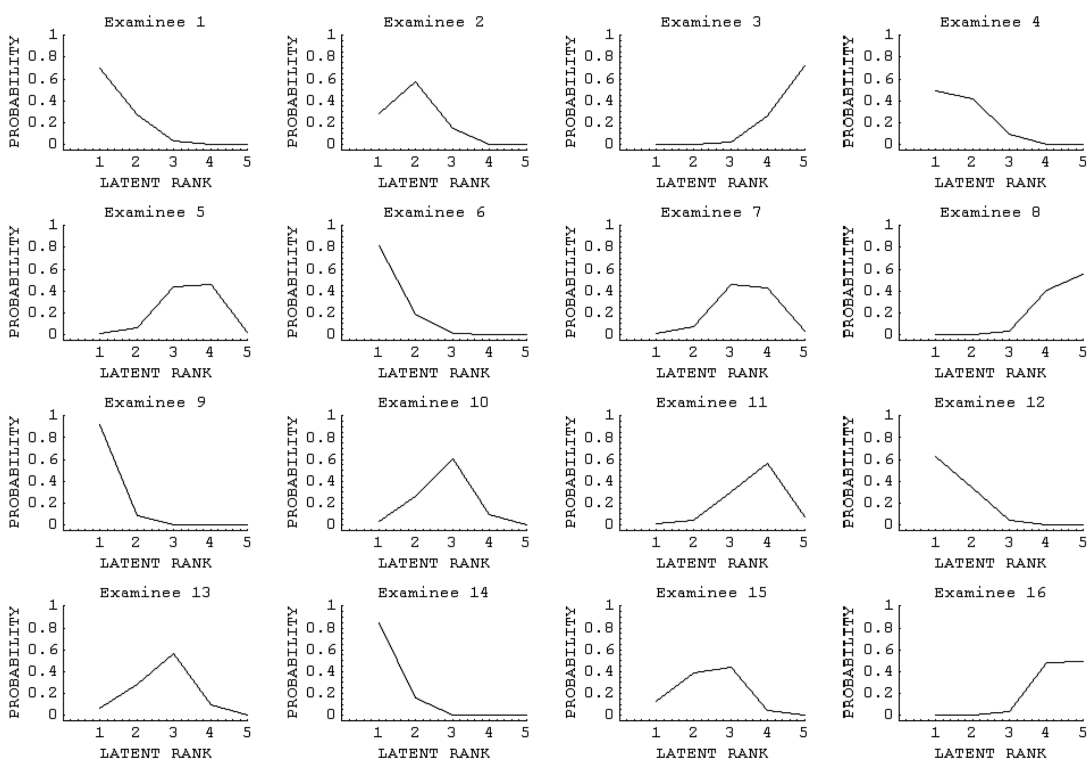
(b) Posterior LRD ($Q = 5$)



(c) Posterior RMD ($Q = 5$)



(d) Scatter Plot ($Q = 5$)



(e) Posterior Rank Membership Profiles of Examinees 1-16 ($Q = 5$)

Figure 4: Figures given by Bayesian Method for $Q = 5$

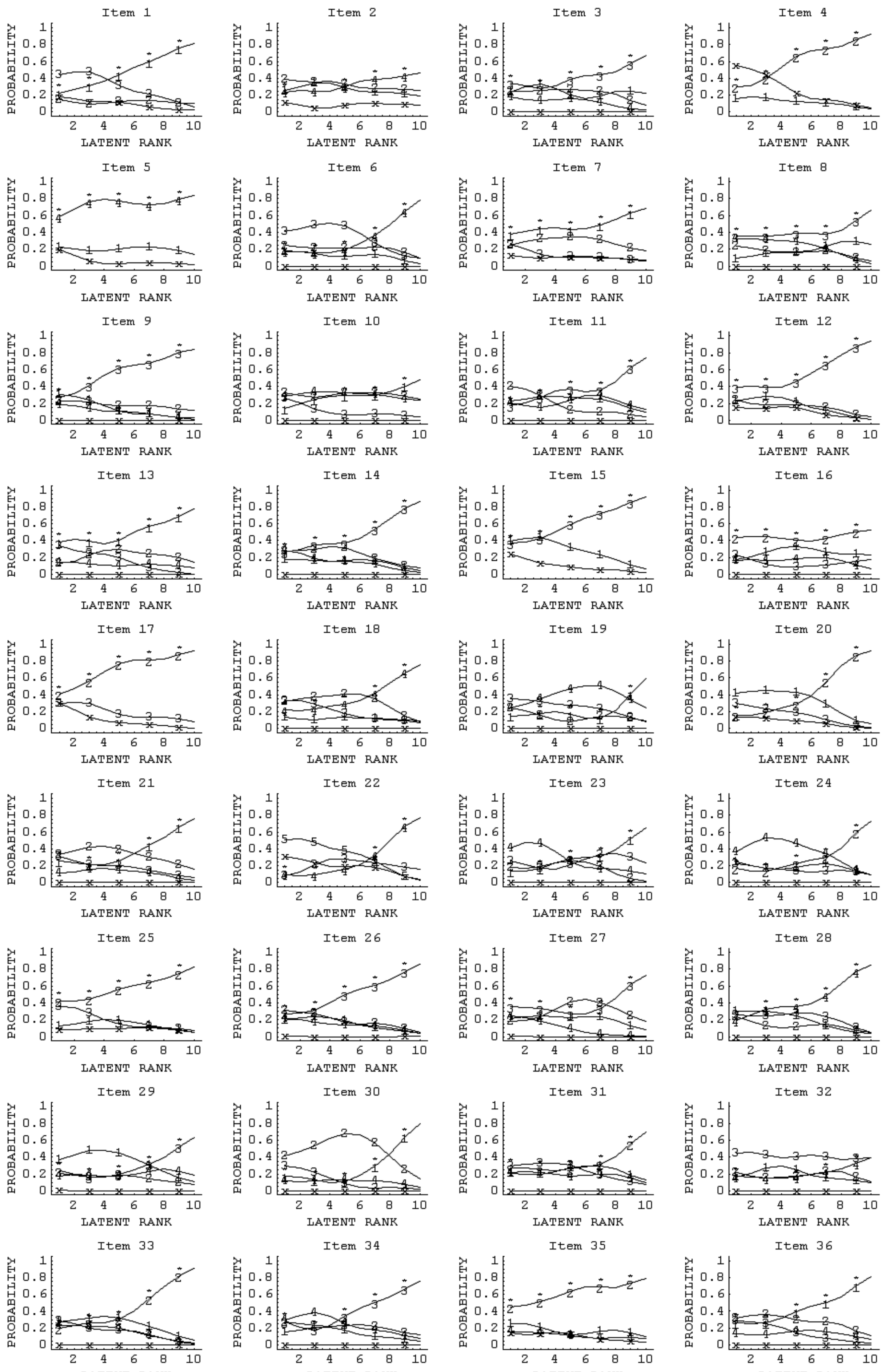
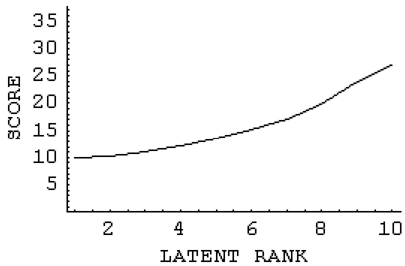
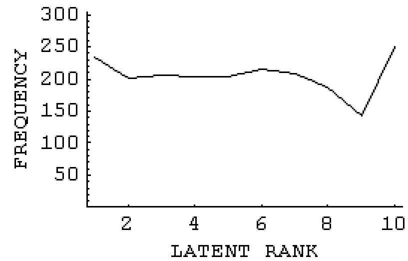


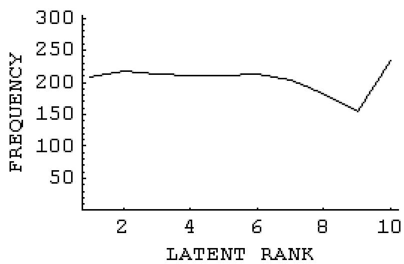
Figure 5: Item Reference Profiles (Trapezoidal Prior Distribution)



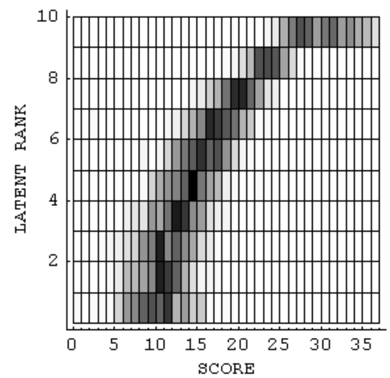
(a) TRP (Trapezoidal Prior Dist.)



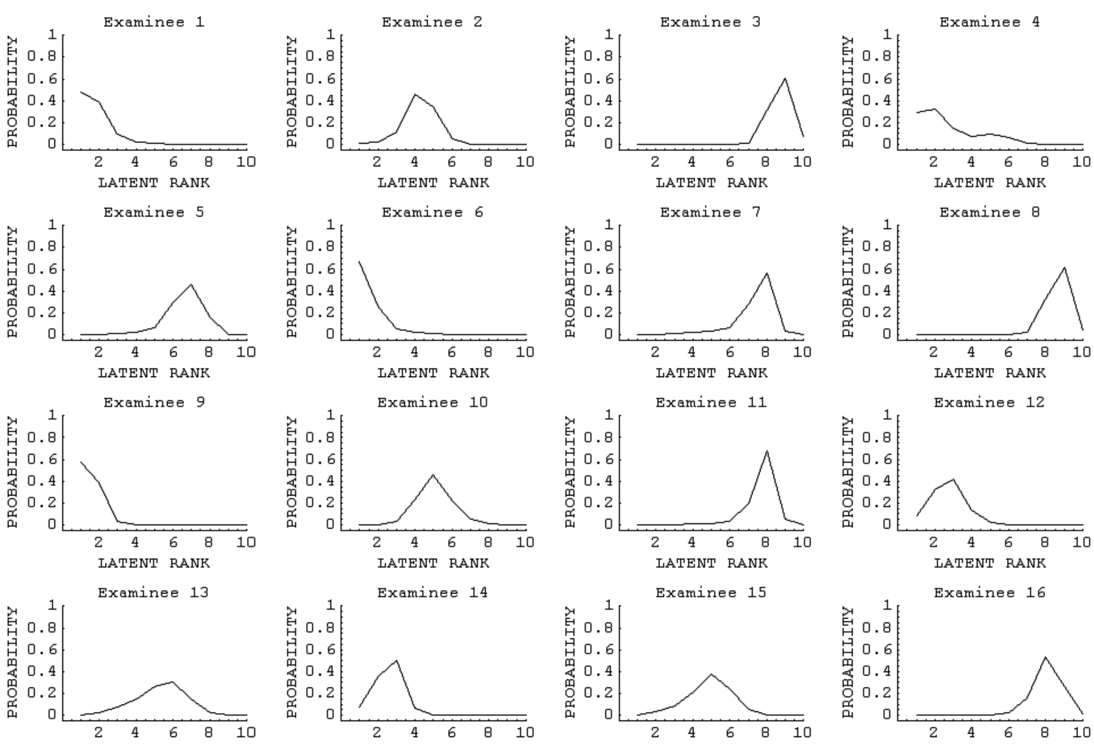
(b) Posterior LRD (Trapezoidal Prior Dist.)



(c) Posterior RMD (Trapezoidal Prior Dist.)



(d) Scatter Plot (Trapezoidal Prior Dist.)



(e) Posterior Rank Membership Profiles of Examinees 1-16 (Trapezoidal Prior Dist.)

Figure 6: Figures given by Bayesian Method with Trapozoidal Prior Distribution

latent rank R_1 is about 0.5 with the Bayesian method, whereas it is around 0.6 with the ML method. In addition, the latent rank of examinee 4 is estimated to be R_2 with the Bayesian method while it is estimated to be R_1 with the ML method.

4 Discussion

We proposed the nominal neural test (NNT) model, which is a polytomous NTT model for analyzing nominal categories data. The NNT model is effective for evaluating the statistical features of correct and incorrect choices. In addition, it reduces to the dichotomous NTT model when the number of items is two, so it is a natural extension of the dichotomous NTT model. Furthermore, it is useful to impose the constraint of monotonicity (Shojima, 2007a, 2007b) on the ICRP of the correct choice when test administrator thinks it natural that the ICRP of the correct choice monotonically increases.

The ICRP of the correct choice is determined not only by the characteristics of the correct choice but also by the characteristics of the incorrect choices. Therefore, it is important to simultaneously review the ICRPs of the correct and incorrect choices to deepen our knowledge about the features or properties of the item.

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